FLUID FRICTION AND CONVECTIVE EXCHANGE IN THIN BEDS OF SPHERES IN A LENGTHWISE FLOW

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Relations are obtained to calculate the drag and convective transfer coefficients in beds of spheres located in a lengthwise flow in narrow annular slits.

Thin beds of spheres are used in a number of heat- and mass-exchange units. In these units, inlet, outlet, and wall phenomena affect the integral characteristics of flow and convective exchange. Examples of such units are selective catalytic reactors or certain types of channel chemical reactors [1, 2]. The available empirical data [1] discusses features of heat and mass exchange in these boundary sections of relatively large (compared to the diameter of the spheres) beds. However, there is not sufficient data in the literature for the case when the sizes of these sections are comparable to the size of the bed and the above-mentioned features become predominant.

We previously studied the porosity of a spherical bed located in a narrow annular slit [3] and convective transfer with transverse flow in thin beds of different thicknesses [4]. The goal of the present study is to investigate thin beds of spheres in a lengthwise flow.

A study was made in [5] of the fluid resistance of a rectangular channel with a bed of spheres for relatively low values of the ratio of channel width to sphere diameter (eight or more). Lower ratios are also of practical interest. Results were reported in [1] from a study of the drag of a cylindrical channel with a bed with the ratio D/d = 1.18-1.43. However, these results cannot be applied to the case of flow in annular slits without first conducting special investigations.

The experimental method used was as follows. A bed of ball bearings of diameter $d = 2.5 \cdot 10^{-3}$ m located in an annular slit between a tube and coaxial rod was exposed to an isothermal descending flow of air (T = 292-295°K). The width of the slit was varied by changing the diameter of the rod. To increase measurement accuracy at low flow rates and reduce the dependence of the drag on the velocity profile at the inlet, we chose to have a fairly long bed — L/d = 100. The ratio of the external and internal diameters of the channel was varied in the range $D_1/D_2 = 1.15$-2.7. The relative hydraulic diameters of the channels and the porosity of the beds are shown in Table 1.

Air flow rate was measured with a diaphragm or a GSB-400 gas meter. Atmospheric pressure was monitored with an aneroid barometer. The pressure gradients on the diaphragm and over the length of the bed were measured with U-shaped liquid-column gauges. Pressure was sampled at 11 points located evenly along the channel $2.5 \cdot 10^{-2}$ m from each other. To average out possible random deviations of static pressure, three holes $0.8 \cdot 10^{-3}$ m in diameter were drilled in the external tube in the measurement section with an angular spacing of 120°. The holes were connected to a manifold.

The equivalent drag coefficient was determined in accordance with [1]:

$$f_e = \frac{2\Delta P\bar{\rho}}{G^2}\,\frac{\varepsilon^3}{a^*}\,\frac{d}{L}. \qquad (1)$$

The specific surface of the wetted surface was calculated with and without allowance for the surface of the walls: $a^* = 6\,(1-\varepsilon)$; $a^*_{wa} = 6\,(1-\varepsilon) + 3/\delta_h$ [1]. The error of f_e for a confidence level $\Omega = 0.95$ was 6-8% in relation to δ_h.

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TABLE 1. Results of Analysis of Tests in the Study of Fluid Resistance (D_1 = 4•10^{-2} m; L/d = 100)

Parameter	δ_h				
	2,12	4	5,96	7,96	10
ε	0,441	0,417	0,419	0,410	0,410
$k \cdot 10^9$, m^2	5,69	5,04	5,40	5,21	6,17
c	0,223	0,209	0,259	0,328	0,428
c_0	66,9	58,6	55,9	53,0	44,4
c_i	0,378	0,304	0,371	0,444	0,529
c_0^{wa}	33,1	39,9	42,8	43,1	37,9
c_i^{wa}	0,226	0,251	0,325	0,400	0,489

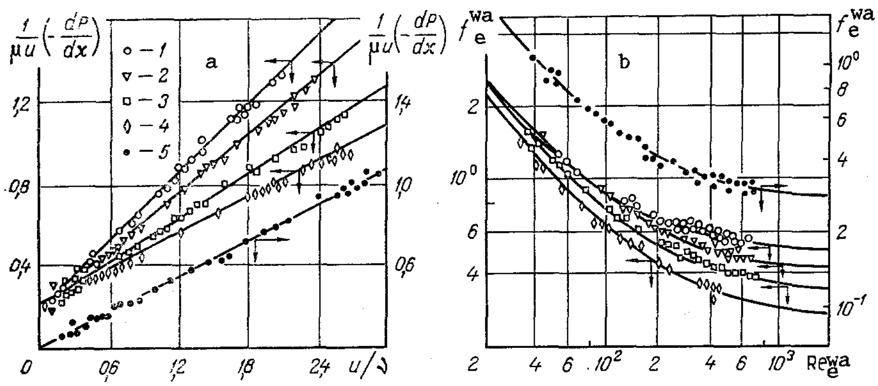

Fig. 1. Dependences: a) of the complex $\frac{1}{\mu u}\left(-\frac{dP}{dx}\right)$ (10^{-9} m^2) on $\frac{u}{\nu}$ (10^{-5} m^{-1}); b) of the drag coefficient on the Reynolds number: 1) δ_h = 10; 2) 7.96; 3) 5.96; 4) 4; 5) 2.12.

TABLE 2. Results of Analysis of Experiments in an Investigation of Convective Exchange (D_1 = 1•10^{-2} m; L/d = 25.8). $Nu_э'/Sc^{1/3} = A\,Re_э^n$

Parameter	δ_h		
	3,87	6,32	10,2
ε	0,49	0,45	0,40
A	0,46	0,504	0,546
n	0,58	0,58	0,58

The nonlinear generalization of Darcy's law in the case of unidimensional flow can be written in the form [5]

$$-\frac{dP}{dx} = \frac{\mu}{k} u + \frac{c}{\sqrt{k}} \rho u^2. \tag{2}$$

Using drag coefficient (1) and Eq. (2), it is not hard to obtain the following binomial relation for f_e:

$$f_e = \frac{c_0}{Re_e} + c_i, \tag{3}$$

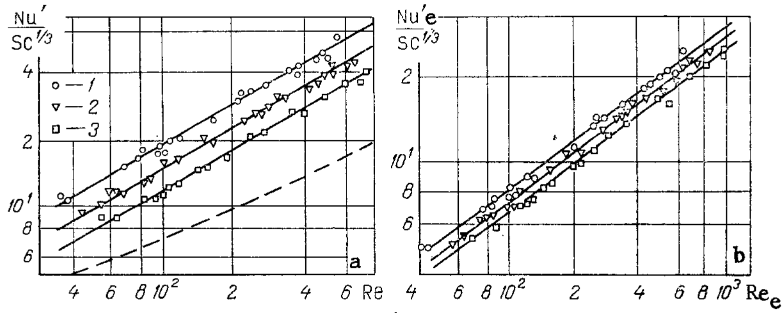

Fig. 2. Dependences: a) of $Nu'/Sc^{1/3}$ on the Reynolds number b) of $Nu'_e/Sc^{1/3}$ on the equivalent Reynolds number: 1) δ_h = 10.2; 2) 6.32; 3) 3.87.

where $c_0 = 8\varepsilon^3 d^2/(a^*)^2 k$ is the coefficient with the viscous term of resistance; $c_i = 2c\varepsilon^3 d/a^* k$ is the inertial term of resistance. For sufficiently large beds of spheres, the constant c entering into the inertial term of resistance c_i is equal to 0.55 [5]. With allowance for the Coseny—Karman equation for the permeability of spheres [5]

$$k = \frac{\varepsilon^3 d^2}{5[6(1-\varepsilon)]^2} \tag{4}$$

the constants c_0 and c_i for the case $L/d \gg 1$, $d/d \gg 1$, and $\varepsilon = 0.38$ are equal to $c_0 = 40$ and $c_i = 0.58$. Thus, the following relation is valid for sufficiently large beds

$$f_e = \frac{40}{Re_e} + 0.58. \tag{5}$$

It should be noted that the inertial term of resistance in the Ergan equation [1] is also equal to 0.58.

We change Eq. (2) to the form

$$\frac{1}{\mu u}\left(-\frac{dP}{dx}\right) = \frac{1}{k} + \frac{c}{\sqrt{k}}\,\frac{u}{\nu}. \tag{6}$$

Having constructed Eq. (6), we can find c and k. The latter quantities unambiguously determine the drag coefficient (see Eq. (3)). The given method was successfully used in [5] for incompressible flow. In practice, the method can also be used for compressible flow if $\Delta\rho/\rho_1 < 0.2$. In this case, dP/dx is replaced by $\Delta P/L$ and the filtration velocity u is referred to the middle section along the channel. The condition $\Delta\rho/\rho_1 \leqslant 0.16$ was satisfied in our experiments.

Figure 1a shows the relations $\frac{1}{\mu u}\left(-\frac{dP}{xd}\right) = \varphi\left(\frac{u}{\nu}\right)$ for different δ_h. It is evident that the relations are linear even when the pressure gradient is not constant along the bed. Table 1 shows the results of a least squares analysis of the data in the form (6). It follows from the table that analysis of the data with allowance for the surface of the walls, as recommended in [1], does not make it possible in the present case to obtain a single relation for different δ_h. However, such an analysis is physically more valid and convenient, since in this case the coefficient c_0^{wa} is nearly constant for different values of δ_h (except δ_h = 2.12) and is equal to about 40.

The results for drag are shown in Fig. 1b, from which it is evident that the data calculated from Eq. (3) (solid lines) agrees well with the test data. One of the equations obtained shows that the drag coefficient is quite dependent on δ_h. Thus, at Re_e^{wa} = 700, a change in δ_h from 10 to 4 leads to a decrease in f_e^{wa} by a factor of 1.8. A similar effect was noted in [1, 5]. This can be interpreted physically as follows. The contribution of the wall sections of the bed to the total drag obviously increases with an decrease in δ_h. Flow in the immediate vicinity of the wall (at a distance d/2 from it) differs from flow in the central region of the bed. In the first case it is evidently structurally similar to flow in channels. This comparison is more valid if we consider that mean bed porosity is 15-20% higher in a wall region of width d/2 than in the center. An estimate shows that in an inertial flow regime such as at $Re_e = 1500$, the drag of a bed of spheres with a porosity $\varepsilon = 0.4$ is roughly one order greater than the drag of a system consisting of a group of cylindrical channels of diameter $D = d_e = 2\varepsilon d/3(1 - \varepsilon)$ and having a total area equal to the area of the through section (cross section) of the bed. Numerical estimates obtained from comparison of flow in the wall region and flow in channels correspond to the change in the drag coefficient with a decrease in δ_h that was obtained in the experiment. This provides grounds for suggesting that the decrease in the drag of beds — more exactly, in the inertial term of drag — with a decrease in δ_h is due to a change in hydrodynamic flow conditions in a narrow wall region of width d/2.

The deviation of the data for $\delta_h = 2.12$ from the general law is evidently due to the ordering of the bed structure. In this case, there is practically only one layer of spheres between the two walls. The walls "organize" the mutual location of the spheres and it becomes more correct to speak of a packing rather than a bed of spheres. According to the data in [1], the drag of such packing may be either higher or lower than the corresponding value for beds of the same porosity.

The measured porosity $\varepsilon = 0.441$ at $\delta_h = 2.12$ is nearly the same as for a packing in which the centers of adjacent spheres lie at the vertices of a regular triangle with a side d. If we continue to compare flow in the wall region with flow in channels, then in the present case ($\delta_h = 2.12$) the channels would be directed at a 30° angle to the tube generatrix and additionally twisted about their circumference. This would lead to a 15% increase in the length of the path travelled by the flow and thus a similar increase in drag. These circumstances make it possible to understand why there is no further reduction in drag at $\delta_h = 2.12$.

The following relation can be recommended for the drag coefficient in the case of lengthwise flow over a bed of spheres ($\delta_h \geqslant 4$)

$$f_e^{wa} = \frac{40}{Re_e^{wa}} + 0.58\,[1 - 1.46\exp(-0.216\delta_h)]. \tag{7}$$

It should be noted that with sufficiently large values of δ_h, Eq. (7) turns into Eq. (5) for sufficiently large beds. Experiments were conducted at $Re_e^{wa} = 20$–700. However, considering that the inertial term can be considered constant at $Re_e^{wa} \leqslant 2000$ [1], the upper limit of applicability of Eq. (7) with respect to the Reynolds numbers can be extended to 2000.

The permeability of beds of spheres can be described by a relation analogous to the Coseny—Karman equation (4) to within 8%, the only difference being that the specific surface is calculated with allowance for the surface of the walls

$$k = \frac{\varepsilon^3 d^2}{5\,[6(1-\varepsilon) + 3/\delta_h]^2}. \tag{8}$$

To find the pressure distribution along the bed, we integrate Eq. (2). Taking the equation of state of the gas $\rho = P\eta/RT$ into consideration, we obtain

$$P_1^2 - P^2(x) = \left(\frac{\mu}{k} + \frac{c}{\sqrt{k}}G\right)\frac{2GRT}{\eta}x. \tag{9}$$

The deviation of the theoretical pressure distribution determined in accordance with (9) from the experimental distribution is no greater than 4% ($\delta_h \geqslant 4$). This comparison shows that Eq. (9) is sufficiently reliable, thus making it possible to determine pressures over the length with an isothermal gas flow. The porosity of the bed for a given δ_h can be found from [3]. Permeability is determined from Eq. (8), and the inertial term c is calculated through c_i^{wa} (see Eqs. (3) and (7)).

In the general case, the problem of integrating Eq. (2) for a nonisothermal flow can be solved numerically by an iterative method. To obtain an approximate analytical expression, we adopt the following assumptions: 1) the temperature distribution is uniform and the gas temperature in a fixed cross section is equal to the temperature of the spheres; 2) the absolute viscosity is independent of pressure, while the temperature dependence is expressed by the equation $\mu = aT^{l}$. It is also necessary to prescribe a law of temperature change lengthwise. We will perform a calculation for the two temperature distributions encountered most often: 1) linear — $T(x)/T_1 = 1 + x(\Theta - 1)/L$; 2) exponential — $T(x)/T_1 = \exp[x\ln\Theta/L] = \Theta^{x/L}$. Using the definition of mean temperature over the length

$$\overline{T}(x) = \frac{1}{x}\int_0^x T(s)\,ds, \tag{10}$$

by integrating (2) we obtain

$$P_1^2 - P^2(x) = \frac{2GRx}{\eta}\,\overline{T}(x)\left\{\frac{\mu[T_{ef}(x)]}{k} + \frac{c}{\sqrt{k}}G\right\}, \tag{11}$$

where $T_{ef}(x) = \left\{\frac{1}{l+l_0}\frac{L}{x}\frac{T^{l+l_0}(x) - T_1^{l+l_0}}{\Delta T\cdot\overline{T}(x)}[\overline{T}(L)]^{2-l_0}\right\}^{1/l}$ is the effective temperature; $\overline{T}(x) = T_1\cdot[1 + x(\Theta - 1)/2L]$, $l_0 = 2$ for a linear temperature distribution; $\overline{T}(x) = LT_1[\Theta^{x/L} - 1]/[x\ln\Theta]$, $l_0 = 1$ for an exponential distribution.

It can be shown that at the limit at $\Delta T \to 0$ Eq. (11) becomes Eq. (9) for isothermal flow. We obtain the following equation from (11) for the total pressure gradient

$$P_1^2 - P_2^2 = \frac{2GRL}{\eta}\,\overline{T}(L)\left\{\frac{\mu[T_{ef}(L)]}{k} + \frac{c}{\sqrt{k}}G\right\}. \tag{12}$$

The drag coefficient for nonisothermal flow can be calculated from Eq. (7) at the temperature T_{ef}. The mean density $\overline{\rho}$ (see Eq. (1)) is determined from the relation $\overline{\rho} = (\rho_1 T_1 + \rho_2 T_2)/2\overline{T}(L)$.

The relation $T_{ef}(L)/\overline{T}(L)$ will be equal to

$$\frac{T_{ef}(L)}{\overline{T}(L)} = \left[\frac{1}{l+2}\,\frac{\Theta^{l+2} - 1}{\frac{1}{2}(\Theta^2 - 1)}\right]^{1/l}\frac{2}{1+\Theta}$$

for a linear temperature distribution and

$$\frac{T_{ef}(L)}{\overline{T}(L)} = \left[\frac{1}{l+1}\,\frac{\Theta^{l+1} - 1}{\Theta - 1}\right]^{1/l}\frac{\ln\Theta}{\Theta - 1}$$

for an exponential temperature distribution.

It is evident from the expressions obtained that the ratio $T_{ef}(L)/\overline{T}(L)$ is no less than unity in magnitude and depends on the amount of preheating of the gas Θ. The value of l ranges within 0.6–0.85 for most gases (H_2, Ar, He, N_2, CO_2). At such values of l, the ratio $T_{ef}(L)/\overline{T}(L)$ is very slightly dependent on l. An estimate shows that the difference of $T_{ef}(L)$ from $\overline{T}(L)$ at $\Theta < 2.5$ is less than 5%, i.e. the total pressure losses with nonisothermal flow can be calculated from the mean temperature of the gas. This conclusion was confirmed qualitatively in [7], where itwas shown experimentally that data on drag with an exponential change in temperature lengthwise ($\Theta \sim 3$) is described well by the relation for isothermal flow at the mean temperature. However, despite the agreement of the total pressure gradients calculated for nonisothermal flow at T_{ef} and isothermal flow at the temperature $\overline{T}(L)$ with $\Theta < 2.5$, the pressure distributions over the length may differ for the two cases in question. If $\Theta > 2.5$, then the thermophysical properties should be referred to the temperature T_{ef}.

We chose a mass-exchange method — specifically, the ion-exchange method — to study convective exchange in thin beds of spheres in a lengthwise flow. The use of this method makes it possible to isolate the convective component and exclude other exchange components (such as conduction and radiation) which complicate methods based on the development of heat flow. The analogy between heat and mass transfer processes taking place under certain conditions [1, 4] makes it possible to use the data obtained for convective heat transfer as well — $Nu_e'/Sc^{1/3} = Nu_e/Pr^{1/3}$. Moreover, the chosen method is relatively simple and quick and is adequately proven (see [7] for example).

The specific method used in our experiment and the unit employed were described in [4]. The sorbent was a monodisperse bed of KU-2 resin with a grain diameter of $0.775 \cdot 10^{-3}$ m, while the sorbate was uranyl-nitrate $UO_2(NO_3)_2$. The concentration of sorbate in the distilled water pumped down through the bed was $2 \cdot 10^{-3}$ kg/m^3, while the pumping time was 20-45 sec. The experimental conditions were such as to ensure external diffusive sorption kinetics. The bed of granules was located in an annular gap between a tube and a coaxial rod. The diameter of the rod was varied. A bed which was relatively long (L/d = 25.8) was chosen to alleviate the effect of inlet and outlet phenomena (including the effect of the inlet velocity profile) on the mean coefficient of convective transfer. Some of the geometric characteristics of the channels are shown in Table 2. As can be seen from the table, the porosity of beds of ion-exchange resin located in narrow annular slits deviates somewhat from the relation $\varepsilon = \varepsilon(\delta_h)$ obtained in [3] for metallic spheres. This is due to features of formation of the structure of beds of ion-exchange-resin granules. These granules may adhere to each other and the wall and form small cavities. However, the method used to analyze the results in equivalent parameters makes it possible to recalculate the data for the case of noninteracting (such as metallic) spheres, even if the porosity of the bed differs some from the porosity of the resin bed.

Ion-exchange sorption occurred on the surface of the resin molecules, but not on the channel walls. This allows us to assume that our tests modeled convective heat transfer in flow through a bed of spheres located between two adiabatic walls. The test results were analyzed in the form of the diffusion Nusselt number determined from parameters of the internal and external models in the form

$$Nu' = \frac{\beta d}{D^*};\quad Nu_e' = \frac{2\varepsilon}{3(1-\varepsilon)} Nu'.$$

The error of the diffusion Nusselt number corresponding to a confidence level $\Omega = 0.95$ was 8-9% for Nu' and 10-11% for Nu_e'.

The ratios $Nu'/Sc^{1/3}(Re)$ for three investigated values of δ_h are shown in Fig. 2a. The solid lines were obtained from analysis of data by the least squares method in the form $Nu'/Sc^{1/3} = A_0 Re^n$. The standard deviation of the test results from the relations obtained was 4.5-6.0%. The dashed line in the figure shows the relation for a single sphere in a free flow recommended by S. S. Kutateladze [1] (with Pr = 1):

$$Nu = 2 + 0.03\,Re^{0.54}\,Pr^{1/3} + 0.35\,Re^{0.58}\,Pr^{1/3}. \tag{13}$$

The empirical data (Fig. 2a) lies considerably above Eq. (13), which indicates an intensification of heat transfer compared to a single sphere in a free flow. It is also evident from the figure that the data is clearly stratified for different values of the relative hydraulic diameter. Thus, the value of Nu' was 56% greater for $\delta_h = 10.2$ than for $\delta_h = 3.87$. However, the relations shown in Fig. 2a pertain to different values of porosity, so we recalculated the data in parameters of the internal flow model (see Fig. 2b). The solid lines in the figure were also obtained from a least squares analysis of data in the form: $Nu_e'/Sc^{1/3} = A Re_e^n$ (Table 2).

It is evident from Fig. 2b that analysis of the results in equivalent parameters did not make it possible to reduce the data on convective transfer in thin beds to a single relation. However, such an analysis diminishes the stratification of the data compared to an analysis using the external flow model (see Fig. 2a). The stratification of the relations $Nu_e'/Sc^{1/3} = \varphi(Re_e)$ for different δ_h cannot be attributed to the experimental error. Rather, it is a reflection of the laws of convective transfer in the media examined. Our data [4] on convective transfer for a fairly large bed L/d = 10, D/d = 12.9 agrees to within 4% with the data obtained here for longitudinal flow in a thin bed with $\delta_h = 10.2$. This means that boundary phenomena have almost no effect on the exchange characteristics of beds in a longitudinal flow in annular slits at $\delta_h \geqslant 10$. It should also be noted that the walls have almost no effect on mean porosity [3] at $\delta_h \geqslant 12$-14. The decrease in the coefficient of convective transfer with a decrease in δ_h can be attributed to the greater velocity of the flow in the wall region and the corresponding reduction in velocity in the central region, which is mainly responsible for exchange. A change in hydrodynamic conditions in the wall region of width d/2 may also have an effect on the mean Nusselt number.

It also follows from comparison of the results on convective transfer in transverse (see [4]) and longitudinal flows in a thin bed of spheres that a change in its thickness differently affects exchange characteristics. With longitudinal flow, the straight lines $Nu'_e/Sc^{1/3} = ARe_e^n$ (in logarithmic coordinates) remain parallel to each other as the thickness (δ_h) increases. With transverse flow, the exponent n increases with an increase in the number of layers in the bed. This difference is evidence of the anisotropy of convective heat and mass transfer in thin beds of spheres.

In conclusion, we note that the data presented here makes it possible to calculate drag, the pressure distribution in isothermal and nonisothermal gas flows, and the coefficients of convective heat and mass transfer in longitudinal flow in a thin bed of spheres. The results obtained can be useful in theoretically studying transport processes in granular media.

NOTATION

a^*, specific surface of the bed of spheres; d, diameter of sphere; D^*, molecular diffusion coefficient; G, mass rate; f_e, equivalent drag coefficient; k, permeability; L, length of bed; P, pressure; R, universal gas constant; u, filtration velocity; s, variable of integration; T, temperature; l, l_0, constants; β, mass-transfer coefficient; $\delta_h = (D_1 - D_2)/d$, relative hydraulic diameter; D_1, D_2, external and internal diameters of the channel; ρ, density; $\Theta = T_2/T_1$, degree of preheating; μ, ν, absolute and kinematic viscosities; ε, porosity; $Re = ud/\nu$, Reynolds number; $Re_e = 4ud/a^*\nu$, equivalent Reynolds number; Nu', Nu, diffusive and thermal Nusselt numbers; Pr, Sc, Prandtl and Schmidt numbers. Indices: 1, 2, at the inlet and outlet of the bed; wa, value calculated with allowance for the surface of the walls.

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